



MARKSCHEME

May 2005

MATHEMATICAL STUDIES

Standard Level

Paper 1

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Paper 1 Markscheme

Instructions to Examiners

Note: The number of marks for each question has been increased to 8. Where there are 2 marks (e.g. M2, A2) for an answer do NOT split the marks unless otherwise instructed.

1 Method of Marking

- (a) All marking must be done using a **red** pen.
- (b) In this paper, the maximum mark is awarded for a **correct answer**, irrespective of the method used. Thus, if the correct answer appears in the answer box, award the maximum mark and move onto the next question; in this case there is no need to check the method.
- (c) If an **answer is wrong**, then marks should be awarded for the method according to the markscheme. (A correct answer incorrectly transferred to the answer box is awarded the maximum mark.)

2 Abbreviations

The markscheme may make use of the following abbreviations:

- M** Marks awarded for **Method**
- A** Marks awarded for an **Answer** or for **Accuracy**
- G** Marks awarded for correct solutions, generally obtained from a **Graphic Display Calculator**, irrespective of working shown
- C** Marks awarded for **Correct** answers (irrespective of working shown)
- R** Marks awarded for clear **Reasoning**

3 Follow Through (ft) Marks

Errors made at any step of a solution can affect all working that follows. To limit the severity of the penalty, **follow through (ft)** marks should be awarded. The procedures for awarding these marks require that all examiners:

- (i) penalize the error when it **first occurs**;
- (ii) **accept the incorrect answer** as the appropriate value or quantity to be used in all subsequent working;
- (iii) award **M** marks for a correct method and **A(ft)** marks if the subsequent working contains no further errors.

Follow through procedures may be applied repeatedly throughout the same problem.

The following illustrates a use of the **follow through** procedure.

Markscheme		Candidate's Script	Marking
\$ 600 × 1.02	<i>MI</i>	Amount earned = \$ 600 × 1.02	✓ <i>MI</i>
= \$ 612	<i>AI</i>	= \$ 602	× <i>A0</i>
\$ (306 × 1.02) + (306 × 1.04)	<i>MI</i>	Amount = 301 × 1.02 + 301 × 1.04	✓ <i>MI</i>
= \$ 630.36	<i>AI</i>	= \$ 620.06	✓ <i>AI(ft)</i>

Note that the candidate made an arithmetical error at line 2; the candidate used a correct method at lines 3, 4; the candidate's working at lines 3, 4 is correct.

However, if a question is transformed by an error into a **different, much simpler question** then:

- (i) **fewer** marks should be awarded at the discretion of the Examiner;
- (ii) marks awarded should be followed by “(d)” (to indicate that these marks have been awarded at the **discretion** of the Examiner);
- (iii) a brief **note** should be written on the script explaining **how** these marks have been awarded.

4 Using the Markscheme

- (a) This markscheme presents a particular way in which each question may be worked and how it should be marked. **Alternative methods** have not always been included. Thus, if an answer is wrong then the working must be carefully analysed in order that marks are awarded for a different method in a manner which is consistent with the markscheme.

In this case:

- (i) a mark should be awarded followed by “(d)” (to indicate that the marks have been awarded at the **discretion** of the Examiner);
- (ii) a brief **note** should be written on the script explaining **how** these marks have been awarded.

Where alternative methods for complete questions are included, they are indicated by **METHOD 1**, **METHOD 2**, *etc.* Other alternative solutions, including graphic display calculator alternative solutions are indicated by **OR**. For example:

$$\begin{aligned} \text{Mean} &= 7906/134 && \text{(M1)} \\ &= 59 && \text{(A1)} \end{aligned}$$

OR

$$\text{Mean} = 59 \qquad \qquad \qquad \text{(G2)}$$

- (b) Unless the question specifies otherwise, accept **equivalent forms**. For example: $\frac{\sin \theta}{\cos \theta}$ for $\tan \theta$.

On the markscheme, these equivalent numerical or algebraic forms will be written in brackets after the required answer. Paper setters will indicate the required answer, by allocating full marks at that point. Further working should be ignored, even if it is incorrect. For example: if candidates are asked to factorize a quadratic expression, and they do so correctly, they are awarded full marks. If they then continue and find the roots of the corresponding equation, do not penalize, even if those roots are incorrect *i.e.*, once the correct answer is seen, ignore further working.

- (c) As this is an international examination, all **alternative forms of notation** should be accepted. For example: 1.7, 1·7, 1,7 ; different forms of vector notation such as \vec{u} , \overline{u} , \underline{u} ; $\tan^{-1} x$ for $\arctan x$.

5 Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy.

There are two types of accuracy error. Candidates should be penalized once only **IN THE PAPER** for an accuracy error (**AP**). Award the marks as usual then write $-1(\mathbf{AP})$ against the answer and also on the **front cover**.

Rounding errors: only applies to final answers not to intermediate steps.

Level of accuracy: when this is not specified in the question the general rule *unless otherwise stated in the question all numerical answers must be given exactly or to three significant figures* applies.

- If a final correct answer is incorrectly rounded, apply the **AP**
- OR**
- If the level of accuracy is not specified in the question, apply the **AP** for answers not given to 3 significant figures. (Please note that this has changed from May 2003).

Incorrect answers are wrong, and the accuracy penalty should not be applied to incorrect answers.

6 Graphic Display Calculators

Many candidates will be obtaining solutions directly from their calculators, often without showing any working. They have been advised that they must use mathematical notation, not calculator commands when explaining what they are doing. Incorrect answers without working will receive no marks. However, if there is written evidence of using a graphic display calculator correctly, method marks may be awarded. Where possible, examples will be provided to guide examiners in awarding these method marks

Throughout the mark scheme follow through is always assumed from one question part to the next unless otherwise instructed.

QUESTION 1

(a) The sixth number is 22 (C1)

(b) $u_{200} = 2 + 199 \times 4$ (M1)(A1)(A1)
 $= 798$ (A1) (C4)

Note: Award (A1) for $a = 2$ stated or used, (A1) for $d = 4$ stated or used.

(c) $S_{90} = \frac{90}{2}(2 \times 2 + 89 \times 4)$ or $\frac{90}{2}(2 + 358)$ (M1)(A1)
 $= 16\,200$ (A1) (C3)

QUESTION 2

(a) $n = 4$ (A2) (C2)

(b) Mean number of games is 9.08 (accept 9). (M1)(A1) (C2)

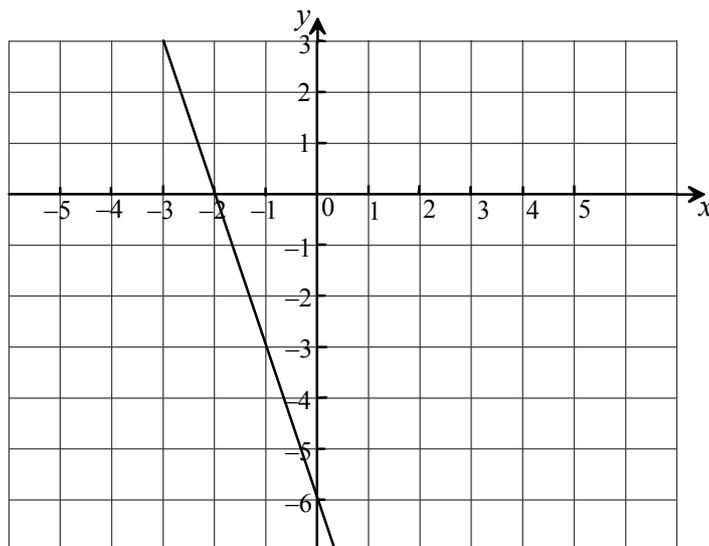
Note: Award (M1) for indicating a sum of games times frequency (possibly curtailed by dots) or for 227 seen.

(c) $\frac{6}{25} \times \frac{100}{1} = 24\%$ (M1)(A1) (C2)

Note: Award (M1)(A0) if 6 is replaced by 10. No other alternative.

(d) Modal number of games is 7. (A2) (C2)

QUESTION 3



- (a) line passes through $(-2, 0)$ (A1)
 line is straight (A1)
 negative gradient (line must be straight for mark to be awarded) (A1)
 correct gradient (line must be straight for mark to be awarded) (A1) (C4)
- (b) $y - 0 = -3(x + 2)$ or $3x + y = 3(-2) + 1(0)$ or $y = -3x + c$ etc (M1)
 $3x + y = -6$ (or equivalent) (A1)(A1)(A1) (C4)

Note: Award (C4) ft for $y = -3x +$ candidate's y -intercept (or equivalent).
 Otherwise award:
 (A1) for y with $=$ in a linear equation,
 (A1) for $y = -3x$ or $y + 3x$ seen or for $m = -3$,
 (A1) for candidate's y -intercept included in a linear expression.
Do not ft candidate's gradient if it is wrong in the diagram,
 no mark for stand alone $-3x$

QUESTION 4

- (a) $0.001673 \times 1000 = 1.673$ ml (M1)(A1)
 $= 2$ ml (nearest whole number) or 2.0 ml (A1) (C3)

Note: Award (M0)(A0) if candidate divides by 1000. The final (A1) ft can be awarded for an answer of 0 ml but **not** for 2×10^{-6} .

- (b) Energy $= \frac{1}{2} \times 351\,223 \times 176.334^2$ (M1)
 $= 5\,460\,407\,707$ (A1)
 $= 5\,460\,410\,000$ ($= 5.46041 \times 10^9$) joules (A1) (C3)
- (c) Units are kg ms^{-1} or equivalent (A1)(A1) (C2)

Note: Award (A1) for any pair correctly presented, (A2) for all 3.

QUESTION 5

(a) $M = 90 \times 2^{\frac{0}{20}} = 90$ (grams) (M1)(A1) (C2)

Note: Award (M1) for $t = 0$.

(b) $t = 80 \dots M = 90 \times 2^{\frac{-80}{20}}$ (M1)
 Therefore, $v = 5.625$ (grams) (5.63 s.f.) (accept either) (A1) (C2)

(c) $45 = 90 \times 2^{\frac{-t}{20}}$ (M1)
 $2^{\frac{-t}{20}} = 0.5$ (M1)
 $t = 20$ years (A2) (C4)

QUESTION 6

(a) $y = x(5 - x)$ or $y = 5x - x^2$ or $25 = c + 5k$ (M1)
 $c = 0, k = 5$ (A1)(A1) (C3)

Note: Award (A1) if no method is indicated but $c = 0$ or $k = 5$ is given alone.

(b) Vertex at $x = \frac{-b}{2a} = \frac{-5}{-2} = 2.5$ (M1)(A1)
 $y = 5(2.5) - 2.5^2 = 6.25$ (M1)(A1)

Note: The substitutions must be attempted to receive the method marks.

Q(2.5, 6.25) (A1) (C5)

Notes: Coordinate pair is required for (A1) but Q is not essential. If no working shown and answer not fully correct, award (G2) for each correct value and (A1) for coordinate brackets. However, if values are close but not exactly correct (eg (2.49, 6.25)) award only (G1) for each less precise value. In this case AP might also apply if number of digits is inappropriate. If differentiation is used, award (M1) for correct process, (A1) for $x = 2.5$, (M1)(A1) or (G2) for 6.25 and (A1) for coordinate brackets.

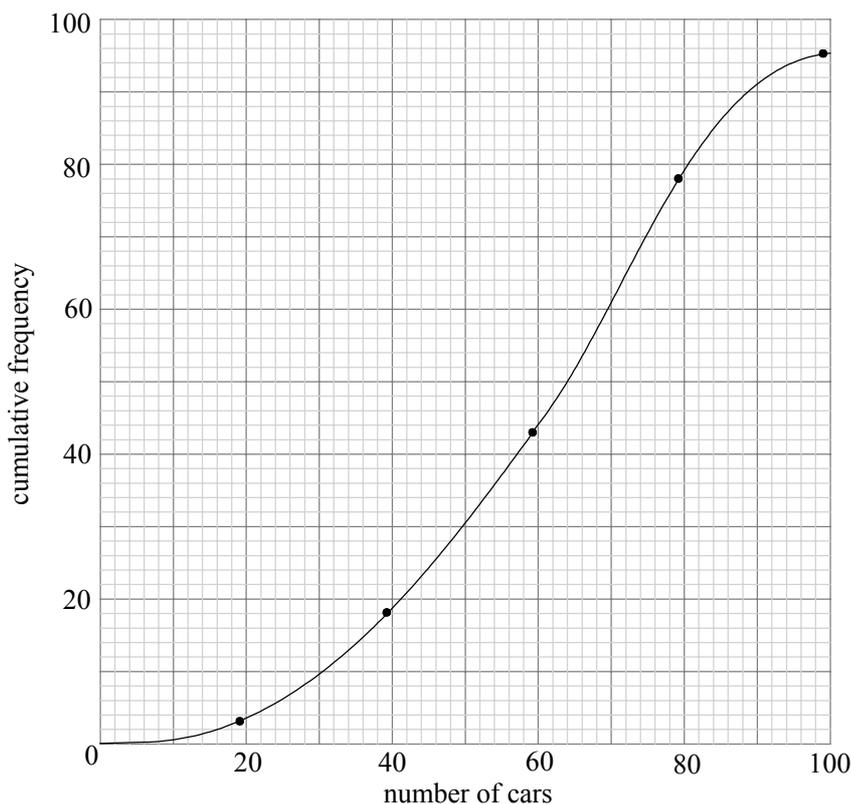
IN QUESTIONS 7 AND 8, EXAMINERS ARE ENCOURAGED TO TAKE EXTRA CARE IN COUNTING THE MARKS ACROSS THE TWO EXAM PAGES. MISSED MARKS OR DOUBLE-COUNTING ARE MORE LIKELY HERE.

QUESTION 7

(a) $w = 43$

(C1)

(b) cars parked per hour



(A5)

(C5)

Notes: Award (A1) for labels (“cumulative” is not essential).
 Award (A1) for scales.
 Award (A2) for all points correct, (A1)(A0) for 4 points correct.
 Award (A1) for neat curve or straight line segments.

The curve must extend to zero for the last (A1).
 The points must be at 19, 39 etc (ft from candidate’s scale).
 If the points are displaced **consistently**, (eg to 20,40 etc or to mid-points) then award (A0)(A1)ft for points.
 If the scale is marked as 19,39 etc, this is allowed as long as 0 is not included, is shown displaced right by one unit.
 Scale marked as an interval (eg 0-19) is acceptable, if written within the interval.
 Bar graph or histogram receive (A0) for curve but if bars are at correct height and terminate at 19, 39 etc or have correct points marked on them, then award (A2) for points or (A0)(A1) ft if points are consistently incorrect.)

(c) median = 63 ± 2

(M1)(A1)

(C2)

Note: Award (M1) for a horizontal line drawn at height 48. Answer must be an integer, follow through from candidate’s graph.

QUESTION 8

(a)

FUNCTION	GRAPH LABEL
$y = a \cos(x)$	C
$y = a \sin(2x)$	B
$y = 2 + a \sin(x)$	A

(A1)(A1)(A1) (C3)

(b) $P = \frac{360}{2} = 180$

(M1)(A1)

or the period is 180 degrees.

(C2)

Note: Award (A1) only for 0–180 or π .

(c) The range is $[2 - a, 2 + a]$ or $2 - a \leq y \leq 2 + a$

(A1)(A1)(A1) (C3)

Note: Award (A1) for each value seen and (A1) for correct [] or $\leq y \leq$.
 $[-a, a]$ (or equivalent) can receive (A2) and $(-a, a)$ or equivalent can receive (A1).

QUESTION 9

(a) $a + b + c = 0$ or any equivalent form

(C2)

Note: The answer **must** be an equation.

(b) (i) $\begin{pmatrix} 5 \\ -2 \end{pmatrix} + \begin{pmatrix} 4 \\ 5 \end{pmatrix} = \begin{pmatrix} 9 \\ 3 \end{pmatrix}$, therefore $c = \begin{pmatrix} -9 \\ -3 \end{pmatrix}$

(M1)(A1)(A1) (C3)

Note: Follow through from answer to part (a)
 Award (M1) for addition of a and b , or $-a$ and $-b$,
 Award (A1) for both values correct,
 Award (A1) for presentation as a column vector.
 Award (A2) for $\begin{pmatrix} 9 \\ 3 \end{pmatrix}$ on its own.

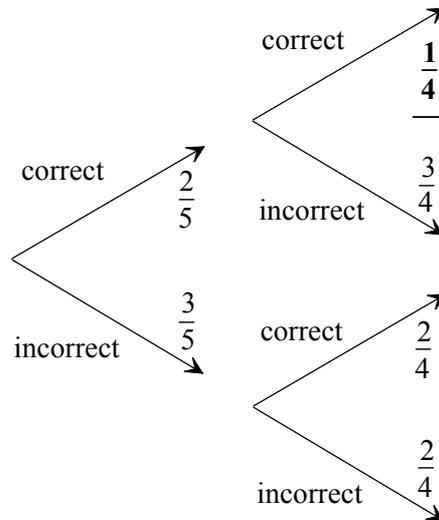
(ii) $|c| = \sqrt{(-9)^2 + (-3)^2} = \sqrt{90}$ (9.49 to 3 s.f.)

(M1)(A1)(A1) (C3)

Note: Follow through from answer to part (b) (i)

QUESTION 10

(a)



(A2) (C2)

(b) (i) $\frac{2}{5} \times \frac{3}{4} + \frac{3}{5} \times \frac{2}{4}$

(A1)(A1)

Note: Award (A1) for each correct product.

$= \frac{12}{20} (= 0.6)$

(A1) (C3)

(ii) $\frac{\frac{2}{5} \times \frac{1}{4}}{\frac{3}{10} + \frac{1}{10}} = \frac{1}{4} (= 0.25)$

(A1)(A1)(A1) (C3)

Note: Award (A1) for $\frac{2}{5} \times \frac{1}{4}$ seen and (A1) for $\frac{3}{10} + \frac{1}{10}$ seen.

QUESTION 11

(a) \$ 4000 / 0.4504 = 8880.99 Ringgit (8880 to 3 s.f.)

(M1)(A1) (C2)

Note: Allow 8881 (nearest Ringgit).

(b) $8880.99 \times (1.0043)^{30} - 8880.99$

(M1)(A1)(A1)(M1)

$= 1230.09$

(A1)

Note: Award (M1) for compound calculation, (A1) for multiplier, (A1) for 30 seen and (M1) for deducting principle.
Use of 1.0043 (no recurrence) is premature rounding: award at most (M1)(A0)(A1)(M1)(A0), but final (A1) ft can be awarded below for 1220.

$= 1230$ Ringgit

(A1) (C6)

Note: Final (A1) is for rounding candidate's answer to nearest Ringgit.
This is awarded only if the interest is calculated.

QUESTION 12

- (a) $A = x^2 + x$ or any equivalent unsimplified expression (A1)(A1) (C2)

Note: Award (A1) for each term.

- (b) $x^2 + x = 30$ or $x^2 + x - 30 = 0$ (C1)

Note: The answer must be an equation.

- (c) $(x - 5)(x + 6) = 0$ or reasonable attempt to use formula. (M1)(M1)

Note: Award (M1) for **both** signs wrong or one error in quadratic formula (if used).

$x = 5$ or $x = -6$ (A1)(A1) (C4)

Note: Award (A2)d for $x = 5$ seen with no other working.

- (d) $x = 5$ because **length** must be positive (must have reason for the mark.) (C1)

QUESTION 13

- (a) $80 \times 62.1 + 2 \times 11 = 4990$ (M1)(A1)

Note: Award (M0)(A0) if 2×11 is subtracted and **ft** the remainder of the question to answers of 61.825 (or 61.8) and 59.7 respectively.

$\frac{4990}{80} = 62.375$ (or 62.4 to 3 s.f.) (M1)(A1) (C4)

- (b) $4990 + 56.8 \times 60 = 8398$ (M1)(A1)

$\frac{8398}{140} = 60.0$ (3 s.f.) (M1)(A1) (C4)

Note: An answer of 60 (2 sf) with no working receives (G2) or with working using 4990 receives (M1)(A1)(M1)(A0) AP, however, if 80×62.4 is used then 60 is an exact answer and can receive all the marks.

QUESTION 14

- (a) $w = (2.6 \times 10^4) \times (5.0 \times 10^{-8})$
 $= 13 \times 10^{-4}$ or 0.0013 (if written as working) (A2)
 $= 1.3 \times 10^{-3}$ (A1)(A1) (C4)

Note: For incorrect answers with no working, award marks as follows:
 13×10^{-4} : (A1)(A1), 0.0013: (G2), 1.3E-3: (G2), 1.3E3: (G1),
 1.3×10^3 : (A2)(A0).

- (b) Statements (ii) and (iv) are incorrect. (A2)(A2) (C4)

Note: Both correct statements given with **at most** one extra, allow (A2).

QUESTION 15

- (a) $K = (12, 16, h)$ or $(12, 16, 22.4)$ (A1)(A1) (C2)

Note: Award (A1) for h as 3rd coordinate, (A1) for 12, 16 **and** parentheses.

- (b) $OD = \sqrt{12^2 + 16^2 + 0^2}$ (M1)
 $= 20$ m (A1) (C2)
- (c) $h = \sqrt{30^2 - 20^2}$ (M1)
 $= 22.4$ m (allow $10\sqrt{5}$ or $\sqrt{500}$). (A1) (C2)
- (d) $\cos \hat{DOK} = \frac{20}{30}$ **or** $\sin \hat{DOK} = \frac{22.4}{30}$ **or** $\tan \hat{DOK} = \frac{22.4}{20}$ (M1)
 $\hat{DOK} = 48.2^\circ$ **or** $\hat{DOK} = 48.3^\circ$ (A1) (C2)
-